

Close Next Wed: HW_6A, 6B, 6C (7.4,7.5,7.7)

See the many practice problems and summary sheets I posted online.

7.4 Partial Fractions (continued)

Aside: These are the types of integrals you will see in section 7.4.

$$\int \frac{1}{2x + 5} dx = \frac{1}{2} \ln|2x + 5| + C$$

$$\int \frac{1}{(x - 4)^2} dx = -\frac{1}{x - 4} + C$$

$$\int \frac{1}{(x + 7)^3} dx = -\frac{1}{2} \frac{1}{(x + 7)^2} + C$$

$$\int \frac{1}{x^2 + 9} dx = \frac{1}{3} \tan^{-1} \left(\frac{x}{3} \right) + C$$

Entry Task: Integrate

$$\int \frac{1}{x^3 + x^2 - 2x} dx = \int \frac{1}{x(x - 1)(x + 2)} dx$$

Step 1: Factor (I did this for you above).

Step 2: Write

$$\frac{1}{x(x - 1)(x + 2)} = \frac{A}{x} + \frac{B}{x - 1} + \frac{C}{x + 2}$$

Do this now:

Find A, B, and C like you did in the worksheet!

Step 3: Integrate

Summary of Partial Fractions

Given a *rational* function $\frac{p(x)}{q(x)}$.

1. If the degree of $p(x)$ is bigger than $q(x)$, divide and simplify.
2. Factor the denominator, $q(x)$.
3. Write the appropriate decomposition:

i) *Distinct Linear Factors:*

$$\frac{x^2 - 3}{x(x - 1)(x + 4)} = \frac{A}{x} + \frac{B}{x - 1} + \frac{C}{x + 4}$$

ii) *Repeated Linear Factors:*

$$\frac{5 + 2x}{(x + 3)(x - 2)^3} = \frac{A}{x + 3} + \frac{B}{x - 2} + \frac{C}{(x - 2)^2} + \frac{D}{(x - 2)^3}$$

iii) *Irreducible Quadratic Factors:*

$$\frac{4x}{(x + 1)(x^2 + 9)} = \frac{A}{x + 1} + \frac{Bx + C}{x^2 + 9}$$

4. Solve for A, B, C Then integrate!

How to integrate

1. Look for simplifications/substitutions
2. Products/Logs/Inverse Trig → BY PARTS
Sin/Cos/Tan/Sec combos → TRIG
Quadratic (under a radical) → TRIG SUB
Rational Function → PART. FRAC.
3. If nothing seems to work, try a substitution.
($u = \text{inside}$, $u = \sqrt{BLAH}$, $u = \text{trig}$, $u = e^x$)